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Research trends in ice mechanics

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Abstract

In sea ice geophysics, the formulation and implementation of a continuum anisotropic ice dynamics model is required in order to increase the spatial resolution of the Polar Ice Prediction System (PIPS) used by the National/Naval Ice Center to provide sea-ice analyses, forecasts, outlooks and ship-routing recommendations within the marginal ice zone of Arctic and Antarctic seas. Currently, too little is known about the formation of leads in the Arctic, a situation that should rapidly improve via automated ice-tracking SAR algorithms. Many questions remain concerning the influences of inhomogeneities (thermal cracks, ridges, thickness variability, and rubble) on wave propagation, constitutive behavior and overall ice strength at various scales. Floe scale ice models appear to offer the means to bridge the scales between geophysical and structural applications by being able to accurately model the mechanics of ridging, rafting and leading. At the scale of ice forces on structures and ships, a diverse range of creep-brittle failure modes awaits incorporation into ice force models. Knowledge concerning the multiaxial compressive failure of freshwater and saline ice is now available. The constitutive modeling of sea ice lags well behind that for freshwater ice. The important issues of scale effects and inhomogeneities on tensile strength at lab- to structural-scale are discussed, as are the links between various scales. © 1999 Elsevier Science Ltd. All rights reserved.

Keywords: Ice mechanics; Ice dynamics; Geophysical scale; Floe scale; Structural scale; Leads; Ridges; Rafting; Breakup; Waves; Ice forces; Ice-structure interaction; Ships; Multiaxial; Grain boundary sliding; Fracture; Compressive; Tensile

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1. Introduction

There has been a long record of support by the U.S. Navy, Army, National Science Foundation, Department of Interior Minerals Management Service and Canadian National Research Council, Canadian Coastguard and Canadian Program on Energy Research and Development for basic research on ice mechanics. Since the late 1960s the offshore industry has also supported a portion of this research as the need to address a variety of Arctic modeling, engineering, and operations problems continued to grow (Gudmestad et al., 1998). The continued development of coupled atmosphere-ocean-ice climate

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models, combined with increasingly powerful computers, has seen the resolution of these models becoming increasingly finer, and the degree and sophistication of the input information more and more detailed (Randall et al., 1998). Until recently, the engineering and operational requirements of the offshore and shipping industry (involving structural dimensions 10–100 m) have essentially remained separate from the requirements of the coupled geophysical models (apparently isotropic models can resolve the ice behavior to the scale of 50 km, although some would argue 10 km). Again, with the rapid increase in computing speed, the distance between these two groups is rapidly diminishing. The stated goals (Curtin et al., 1995) of a recent Sea Ice Mechanics Accelerated Research Initiative (SIMI) organized by the U.S Office of Naval Research between 1991 to 1996 reflected the present needs of both of the above groups. That is, understand sea ice constitutive laws and fracture mechanics over the full range of geophysical scales, and determine the scaled responses to applied external forces and develop physically based constitutive and fracture models.

The field of ice mechanics is comprised of research into the mechanical, constitutive and fracture properties of a range of different ice types. Applications include ice forces on ships and structures, ice dynamics in the polar regions and global climate models, and shipping forecasts. This field has been established over the last five decades (Korzhavin, FR62; Coon et al., FR74; Michel, FR78; Coon et al., FR81; Goldstein, FR83; Hallam et al., FR87; Jones et al., FR91; Richter-Menge, FR92; Dempsey et al., FR93; Dempsey and Rajapakse, FR95; Langhorne, FR96; Curtin, FR98; Shen, FR98; Weeks, F98; Weeks and Timco, FR98). The 'Further reading' section is illustrative of the progress achieved. The notation FR# denotes the Further reading section and the year of publication. Many of the FR papers are provided to help the reader gage the progress of ice mechanics; by understanding the history of the subject more fully, perhaps the research trends are more obvious. Readers will find that the glaciological, ice physics, and ice chemistry literature has not been included.

The first portion of this paper discusses ice mechanics at the geophysical scale, floe scale, structural scale, and then discusses various links between scales. The final portion lists research trends, proceeding (generally) from structural scale on up to planetary scale.

2. Geophysical scale modeling

Arctic sea ice is in constant motion, driven mainly by wind stress. Leads form under conditions of divergence; rapid refreezing of the open water creates thin lead ice (leads are large open fissures in pack ice or on a single floe that form as the result of deformation processes and spatial variations in ice thickness and strength). Subsequent convergence or shear causes the leads to be crushed to create pressure ridges, semi-permanent features of the ice cover which contain about half of the total Arctic ice volume (Wadhams, 1995). The formation of leads in the arctic has been a cause for much study. Leads characteristically form long, narrow openings, meters to hundreds of meters wide and kilometers long. Lead systems can span hundreds of kilometers to a 1000 km or more. Leads are of fundamental importance to the overall Arctic heat budget, to ship navigation, ice production, and optimal submarine paths.

Accurate descriptions of the geophysical motions of sea ice (often called sea ice dynamics) require the development of physically based constitutive and fracture models at the scale of many tens of kilometers. A widely applied ice dynamics model was developed by Hibler, FR79; Untersteiner, FR86 based in large part upon the AIDJEX ice dynamics model (Coon et al., FR74; Pritchard, FR81). Hibler's model implicitly assumes that a description of the complex sea ice medium as a two-dimensional continuum is possible. A viscous-plastic constitutive law is included which relates the ice deformation and thickness to the internal stresses in the ice cover. The ice strength is empirically defined as a function of the ice thickness and fraction of open water. Typically, the large-scale ice dynamics

Table 1
Hierarchical Structure for Sea Ice

Spatial Scale	Temporal Scale (days)	Stress rate	Strain rate	Features
100 km	100–500	1–100 kPa s^{-1}	10^{-6} to 10^{-8} s^{-1}	pack ice
10 km	10–100			floe aggregation
1 km	1–10			Floe, lead. ridge
1–100 m	< 1			thermal cracks finger rafts melt ponds macrocracks
1 cm–1 m		1–10 MPa s^{-1}	10^{-2} to 10^{-5} s^{-1}	crystals (aligned) brine pockets brine channels microcracks

models use a reduced thickness distribution of two thicknesses: open water and thick ice (Thorndike et al., FR75). Flato and Hibler (1995), Stern et al. (1995), Steele et al. (1997) and Lemke et al. (1997) provide an update of this model.

Overland et al. (1995) have examined sea ice mechanics from the viewpoint of subsets of processes based on scale and their interaction with adjacent scales (see Table 1). They observed small regional scale processes at the scale of 10 km and discontinuities in the velocity or stress state along well-defined boundaries (Stern et al., 1995). They argued that these features can affect the larger-scale sea ice distribution and dynamics at the scale of 500 km. Atmospheric forcing and sea ice deformation have matching energetic scales at several hundred kilometers and time scales of days. Thermally induced ice stresses are low frequency (order of days) while ice motion induced stresses have both high-frequency (order of hours) and low frequency components (Richter-Menge, 1997).

The ice strength is a quantity that lies at the core of geophysical scale modeling. Often not reported, it is deduced by comparing predictions of ice concentration and drift to data collected from drifting buoys, satellite imagery, and submarine-based ice thickness profiles. Such estimates of the compressive strength of sea ice at the scale of 100 km give strength resultants of 1–100 kN/m. The 'geophysical' strength is limited by the formation of leads, rafting and ridging (Rothrock, FR75). Direct measurements of pack ice stresses, obtained at floe scale (which is not well defined, since this scale depends on the thin-to-thick ice composite integrity) and often in the vicinity of active ridging, provide estimates that are an order of magnitude larger.

The importance of reliable ice dynamics models is tied to the mission of the National/Naval Ice Center (NIC). The mandate of the NIC is to provide sea-ice analyses, forecasts, outlooks and ship-routing recommendations within the marginal ice zone of all Arctic and Antarctic seas, with the support of the U.S. Navy, NOAA and the U.S. Coastguard. A necessary and critical element of that support is the Polar Ice Prediction System (PIPS), an operational numerical model used and maintained by the Fleet Numerical Meteorology and Oceanography Center (FNMOC). The PIPS 2.0 model, which covers the entire Arctic area of interest, uses a coarse spatial grid, each element approximately 127 km on a side (Cheng and Preller, 1996). A new version (PIPS 3.0) is under development, and it is intended that this version have an improved spatial resolution of 10 km or better; the anisotropic ice dynamics model will be a key component of PIPS 3.0. The actual resolution (both in time and space) of these ice dynamics models is a key topic for consideration. While a lot of research has concentrated on constitutive laws for ice dynamics, the localization of deformation (the formation of leads, ridges, shear bands) has not been modeled explicitly. Some form of nonlocal approach or fracture model with softening stress-displacement laws for normal as well as shear components may be required. In other words, a suitable ice dynamics model must involve a material length. As is discussed in the next section, such a model may well involve a discrete element (random particle) approach. In the latter case, the size of the elements cannot be arbitrarily large. The size of the elements (aggregates of the floes) imposes a particular characteristic length, with an associated degree of localization. The particle size controls the resolution that can be attained.

3. Floe scale models

Eventually, ice dynamics models will have to accurately model the floe scale failure processes of leading, ridging and rafting. The ultimate scale of resolution rests on how much of the physics of the lead formation and interaction and ridging processes can be woven into a continuum framework. For instance, Tremblay and Mysak (1997) incorporate dilatancy into the Hibler-type constitutive law via a double sliding granular flow model (Balendran and Nemat-Nasser, 1993). The opening of leads associated with shearing deformation is therefore an outcome of this formulation. However, such models treat the often-fragmented granular mass as a continuous medium. The latter approximation is reasonable if the extent of ice being modeled is more than ten floe diameters (Overland et al., 1998), but only if the medium is uniformly cemented by lead ice, deforming uniformly and quasi-statically (Lewis and Richter-Menge, 1998; Dempsey et al., 1999). The converse approach is to formulate a discrete element or particle approach (Hopkins, 1994, 1996, 1998; Jirásek and Bažant, 1995). Ultimately, one might be able to integrate a high resolution discrete element sea ice model into the PIPS 3.0 large-scale sea ice model by, for instance, replacing one or more cells in the large-scale ice model. The large-scale model would furnish the boundary conditions needed to drive the discrete element model. The discrete element model would facilitate the high-resolution simulation of areas of interest in the central arctic or in coastal regions. The formulation of a truly floe scale model for sea ice, and an improved understanding of the actual spatial resolution of the competing continuum ice dynamics models, are now emerging as important research topics at the floe scale and above. On the basis of AVHRR imagery, drifting buoys with GPS navigation, and SAR motion vectors, the motion of large areas of sea ice can be analyzed (Overland et al., 1998). The Beaufort sea ice has been observed to move in 20–150 km rigid aggregates, separated by linear deformation zones or sliplines. The limiting spatial resolution of the isotropic continuum ice dynamics models is thought to be of order 10 km. This is based on the width of the observed slip lines and the fact that recent SIMI stress measurements were better correlated with buoy (displacement) measurements at the 10 km scale than at the 5 km scale (Richter-Menge et al., 1996). The essential importance of a floe scale model of the Arctic ice pack is the ability to model failure processes at the scale of individual ice parcels. The variable thickness of the Arctic ice pack causes the formation of thick ice through ridge building and thin ice through the nucleation of leads. These various pieces of ice may freeze together, overlap to raft or form pressure ridges, split or separate to form leads, depending on the prevailing conditions. Present day discrete element or particle models can already explicitly simulate accurately an assembly of thousands of ice parcels with a distribution of sizes, shapes and thicknesses: the anisotropy is inherent. Moreover, as computer power increases, so can the scope and detail of these models. An understanding of the energetics of pressure ridging is seen as critical to the determination of ice strength on a geophysical scale (Rothrock, FR75; Pritchard, 1992; Hopkins, 1998). In addition, the pressure ridging of first-year ice that is forcing a thick multiyear floe into contact with a structure is a load limiting deformation mechanism as regards ice loads on offshore structures (Blanchet, 1990; Blanchet, 1998). Ridge formation is of central importance, because the amount of horizontal load an ice sheet can transmit is often limited by ridging. The floe scale modeling approach appears to have the potential to significantly advance knowledge both at the geophysical scale and at the structural scale. Should this be true, a framework for the much-desired linkage between vastly different scales will be set in place. Such an approach needs to be validated by independent predictions of measured quantities, both upscale and downscale. For instance, consider a downscale application. It has long been asked of the ice dynamics community to consider the limited driving force concept used to estimate ice forces at the structural scale (Croasdale, FR88). Up to the present time, however, the isotropic continuum models developed to date cannot 'fix a point' in the mesh. This just leads to a singularity. If a floe scale model is developed, then the model should have the capability to 'fix a floe' and thus estimate the ice force that might be imposed on a structure.

4. Structural scales

It is interesting to note that nearly three decades ago, Weeks and Assur, FR72 stated: '*We feel that an understanding of the scale effect in ice testing is essential before a thorough scientific basis can be developed for the utilization of small-scale testing in engineering design problems.*' An understanding of the scale effect is still being sought, primarily because large scale testing is fraught with difficulty and is also very expensive. Not many experiments can be done in any one program, and few field programs have been accompanied by the requisite subsidiary data,...'*for a good reason: The collection of this type of information is time consuming*' (Weeks and Assur, FR72).

The design of offshore structures and marine transportation systems, principally for petroleum exploration and production, has driven much of the ice mechanics research over the last three decades to be focused on a physical range in scale from a fraction of a meter to several hundred meters. Much of this ice mechanics research has focused on the interaction of intact ice sheets, ridges, rubble fields, and fragmented ice covers with fixed and moving structures (Croasdale, FR88). In the literature concerning ice forces on structures, the effective pressure is defined as the total interaction force divided by the contact area, which is usually taken to be the product of structure width and thickness. The measured effective pressures versus contact area show a large range in the data from small-scale tests (1–20 MPa), whereas the data from full-scale measurements reveal a dramatic decrease with increasing nominal contact area (Sanderson, FR88; Blanchet, 1990; Masterson and Frederking, 1993; Blanchet, 1998).

Recently, there has been the release of important data gathered in the Canadian Beaufort Sea on the interaction of different ice conditions with wide offshore structures (Wright and Timco, 1994; Blanchet, 1998). This data has shown that first-year ice loads are on the order of 100 MN, with the highest recorded loads about 400 MN due to the crushing failure of a large multi-year ridge. As more of this data becomes available, it will greatly improve the understanding of ice forces. To ensure that the data is maintained, the NRC in Ottawa has collected all of this ice load information from the Beaufort Sea (Timco, 1996). There is considerable interest, on the part of the design engineer, in determining just how far ice loads can be lowered, safely. Croasdale (1997) emphasizes misgivings on the part of design engineers in being able to make direct use of current indentation theories and small-scale ice properties. As stated by Croasdale (p.66, Curtin et al., 1995): '*To date, however, the engineer's view is that no methods based on the physics of the material appear to be usable in a practical sense to predict ice crushing pressures against structures.*' Essentially, the design ice force has always been much less than small-scale strengths would predict (Blanchet et al., 1997). There is the need for an increased recognition of, and emphasis on, both scale and velocity effects as regards the topic of ice forces on structures. The development of probabilistic failure models, for both local and global loads, will proceed rapidly (Blanchet, 1990; Nessim and Jordaan, FR91), once estimates for these local and global loads harmonize. Recent papers on the development of design codes for ice-strengthened ships to be used in the Arctic or the Baltic (Kujala, 1994; Brown et al., 1996) have used local pressure-area curves, based on a database of damage statistics gathered and analyzed to evaluate the extreme ice load levels for ships. The NRC in Ottawa has also developed an extensive database related to ship damage in different ice conditions (Timco and Morin, 1998). The latter database contains over 1400 interaction events of ships in a variety of ice conditions. Approximately 20% of these events relate to vessel damage.

5. Links between different scales

The tensile fracture mechanics of sea ice, from lab- to structural-scale, has now been established (Mulmule and Dempsey, 1998). Scale effects on the tensile strength over the size range of 0.1–100 m have now been measured, and geophysical tensile strengths are predicted at or less than the scale of 1000 m, depending on the effective crack lengths one expects at these larger scales. If one adopts the viewpoint that scale effects in the compressive strength of sea ice are but a reflection of the scale effect on the tensile strength, one would expect the geophysical stress resultants to also become scale invariant at length scales of a similar order. Certain constitutive quantities examined at both lab- and structural-scale reveal no scale effect, so it is clearly time to obtain a greater understanding of the scale-invariant versus scale dependent parameters.

A key task that is frequently discussed but never done, apparently, is to establish the spacing of thermal cracks (at least in one location), and to excavate and determine their depths and shapes. Thermal cracks, the thickness variability, the brine channel networks, the inherent heterogeneity in strength are all factors that may establish a transition in scales above 100 m.

When ice sheets interact with structures the failure (compressive) strength displays a pronounced scale-effect (Sanderson, FR88; Blanchet, 1998). Plots presenting peak strengths regardless of such factors as failure mode, ice type, and loading rate have been in existence since the early seventies. Sanderson's plot is the most well known. There is now no doubt that a family of pressure-area or pressure-width curves will prove to be necessary, depending on the application, ice type, ice velocities, etc. (Masterson and Frederking, 1993; Brown et al., 1996; Blanchet, 1998). One real difficulty, however, is that this pressure-area/width dependence, while so important, depends on peak pressures under compression. Compressive failure strengths, however, depend on the boundary conditions and scale. While a rather large number of failure mechanisms and loading situations must be analyzed, it does not seem rational to try to deterministically model every brittle mode of failure.

At the geophysical scale, the quantity (Steele et al., 1997; Pritchard, 1998b) most relevant to ice strength is inferred by the comparison of modeled and observed buoy drift. The actual medium is assumed to be incapable of sustaining tensile stresses, and yet the single most reliable index appears to be the limiting tensile strength of the pack ice. The most direct path to linking the structural and geophysical scales is in all likelihood through the floe scale models. Stern et al. (1995) show a 75 km by 500 km mosaic of ice motion over the space of three days and reveal shear bands separating large blocks of ice moving as one. Immediate analogies regarding the mechanisms of dilatancy and failure of dense granular soils come to mind. With the rapidly growing capabilities of automated ice-tracking SAR algorithms (Stern et al., 1995), and the ability to model fracture and breakup on the scale of the ice thickness and floe scale (depending on the failure modes), it should soon be possible to quantitatively link ice knowledge at different scales. The shape and size of the failure envelopes for the strength of ice is a topic that is currently being queried at vastly different scales (Hopkins, 1996; Hibler and Schulson, 1997; Lemke et al., 1997; Melton and Schulson, 1998; Coon et al., 1998) and for dramatically different states of stress. In this context, it is advisable to remember that multiaxial lab-scale experiments have been conducted on cubes or cylinders of dimension 15 cm or less (the cubes being proportioned 1:1:1), whereas the regional grid of 10 km x 10 km x 2 m thick is proportioned at 1:1:0.0002.

6. Research topics

6.1. Ice-structure contact zone

At the present time, the relative importance of contact area, ice sheet velocity, and the structural width versus ice thickness ratio on the global design ice force is not sufficiently well understood (Määttänen, FR78; Goldstein and Osipenko, FR83; Ponter et al., FR83; Palmer et al., FR83; Evans et al., FR84; Timco, FR86; Daley, 1992; Johnston et al., 1998; Daley et al., 1998). Once gained, this knowledge needs to be incorporated in ice-structure interaction models (Riska, 1995; Sodhi, 1995; Kärnä et al., 1998) and global ice load models (Blanchet, 1990, 1998; Shkhinek et al., 1994; Kärnä and Rim, 1996). A very important question concerns the role of brittle failure processes in reducing effective pressures at higher ice sheet velocities versus the dependence on contact area implicit in the Sanderson plot, or versus width (Blanchet, 1998). Important sub-problems include the nucleation of cleavage cracks, spalling and flaking, the size and failure of individual crushing zones, and the ductile-to-brittle transition speed and its dependence on temperature. The hypothesis that the effective pressure generated during ice-structure interaction depends primarily on the contact area is not universally accepted: the influence of ice velocity (or indentation speed) and surrounding ice extent is perhaps just as important (Saeki and Ozaka, in Tryde, FR80; Blanchet, 1990, 1998; Sodhi, 1998a, 1998b; Sodhi et al., 1998; Masterson et al., 1999).

The contact zone between a structure and ice during indentation is now being studied intensively. The contact is comprised of direct ice/structure contact and indirect contact through a layer of crushed ice. High pressure zones (also called ‘hot spots’ or ‘critical zones’) have been measured and are associated with nonsimultaneous crushing (Kry, FR78; Kry, FR81; Glen and Blount, FR84; Ashby et al., FR86; Joensuu and Riska, 1989; Fransson et al., 1991; Riska, FR91; Riska, 1991; Jordaan et al., 1993; Tuhkuri, 1995; Sodhi, 1998a). The formation of these concentrated pressure spots occurs regardless of the scale or type of interaction, although the associated spatial density is influenced by confining pressure and scale effects (Johnston et al., 1998). For thin ice sheets a line-like array of high pressure zones occurs, but this is not the case for thicker ice sheets. For ship impacts, cases of advancing contact (Frederking et al., FR90), and thick ice features (ice islands and icebergs), it may be preferable to consider a random hot spot formation-location process over the central portion of the contact area (Jordaan and Xiao, 1998).

During indentation at low speeds, creep deformation of ice in the region of the line-like contact leads to expansion of the contact area, reduction of the interfacial pressure, and simultaneous contact across the width of the indentor. At these speeds, one or several cleavage or spalling cracks form in the plane of the ice sheet at various depths (Hirayama et al., FR74; Kry, FR81; Zou et al., 1996; Tuhkuri et al., 1997). In most experimental observations of the ductile-brittle failure process in the contact region, a certain proportion of the contact occurs through a layer of crushed ice. This has led to studies of the dynamics of the ice crushing process and its associated extrusion (Nevel, FR86; Jordaan and Timco, FR88; Tunik, in FR91). The importance of the crushed layer has was first realized through medium scale field indentation experiments (contact areas ranging from 0.2 to 0.5 square meters, Frederking et al., FR90) and then at the scale of offshore structures (Jeffries and Wright, FR88). This has led to an intensive study of the constitutive properties of crushed ice (Sayed and Frederking, 1992; Savage et al., 1992; Singh and Jordaan, 1997; Jordaan et al., 1998). It has been hypothesized that the systematic fluctuation in loads in the critical zones is a result of softening of the ice under triaxial confinement and high shearing stresses (Jordaan et al., 1998); mechanisms discussed include microcracking (shear banding), recrystallization and grain boundary melting. Palmer and Sanderson (1991), Parsons (1991), Tuhkuri (1994) and Bashkirov and Vityazev (1996) have examined the formation of crushed ice as a fragmentation process. The latter two papers offer criticisms of the earlier work and found a power law particle size description inadequate. Parsons (1993), who suggested that fragmentation processes in ice may be scale dependent, also reached the same conclusion.

6.2. Constitutive behavior under compression

The historical development of constitutive equations for the laboratory-scale deformation of freshwater granular and columnar polycrystalline ice is provided by the following papers and their reference lists: Duval, FR78; Sinha, FR78; Gold and Sinha, in Tryde, FR80; Ashby and Duval, FR85; Sunder and Wu, FR89a; Sunder and Wu, FR90; Duval et al., in FR91; Jordaan and McKenna, in FR91; Sinha, in FR91; Zhan et al., 1994. These models are not directly applicable to sea ice due to significant microstructural differences between freshwater and saline ice (Schapery, 1997a; Cole and Shapiro, 1998). A physically based model for the anelastic response of sea ice based on a dislocation and grain-boundary relaxation mechanisms has been formulated (Cole, 1995; Cole et al., 1998). A phenomenological approach based on nonlinear viscoelasticity/viscoplasticity has also been adapted from previous developments in polymers and composites (Schapery, in FR91; Schapery, 1997b; LeClair et al., 1999).The same type of approach viscoelastic/viscoplastic constitutive model for polycrystalline ice with distributed damage has been developed (Xiao and Jordaan, 1996; Stone et al., 1997; Jordaan et al., 1998).

6.3. Multiaxial compressive failure

Intensive efforts have been devoted to the multiaxial compressive failure of polycrystalline freshwater and saline ice (see the reviews in Lainey and Tinawi, FR84; Rist, 1997; Schulson, 1997). Topics researched include: the fracture and friction of ice under conditions of multiaxial compression at rates and temperatures for which the deformation behavior is predominantly brittle-elastic; the multiaxial strength envelopes combined with phenomenological descriptions of the cracking and faulting mechanisms; and the damage and recrystallization processes in ice (Kalifa et al., 1992; Xiao and Jordaan, 1996; Stone et al., 1997; Melton and Schulson, 1998; Jordaan et al., 1998). Mechanisms discussed include across-column cracking, grain boundary sliding (Elvin and Sunder, 1996; Gupta et al., 1997), and frictional sliding. Traditionally, the lab-scale compressive tests have been conducted for relatively slow strain rates; there is an increased recognition that strain rates in the field can be appreciably higher. Recrystallization at higher strain rates and higher confinement is more significant than anticipated. The models put forward by these investigators rely heavily on the seminal work completed by Horii and Nemat-Nasser and Ashby and Hallam (Schulson, 1997). There is not universal agreement that wing cracks control the brittle compressive failure of ice (Nixon, 1996). The wing crack model has recently been extended (Schulson et al., 1999) to incorporate the development of ‘splay fractures’ (Cooke, 1997) from one side of the sliding parent crack; the latter fractures provide an instability-triggering mechanism.

6.4. In-situ compression tests

Full thickness *in-situ* uniaxial compression experiments have been attempted (see the references listed within Chen and Lee, FR86). Full thickness test blocks sized at 3.05 m times 6.10 m (with ice thicknesses growing from 1.2 m to 1.8 m during the test program) were compressed at stain rates ranging from 10^{-7} to $10^{-5}\,\mathrm{s}^{-1}$. Small blocks of ice were harvested for lab-scale uniaxial compressive tests. Much has been made of these tests, generally to support scale-invariance arguments. However, vertical splitting (or slabbing) cracks that had formed along the length of the ice block were observed in all tests. Not surprisingly, at peak pressures, the transverse strains equalled or exceeded the longitudinal strains. The ice strengths that should have been reported were the strengths associated with the nucleation of the slabbing cracks. Moreover, to argue scale effects (for or against), one must have varied the scale of the tests significantly and self-similarly (in plan, in this case), and all of the tests need to be carried out under the same conditions.

6.5. *Macroscopic compressive failure*

Most papers in ice mechanics dealing with compressive failure of ice have been focused on the microscale processes for incipient damage processes, such as micromechanical models for the nucleation of microcracks in polycrystalline freshwater ice, whether this be due to elastic anisotropy, porosity, or grain boundary sliding (Wu and Niu, 1995; Wu and Zhang, 1995; Elvin and Sunder, 1996; Gupta et al., 1997; Weiss and Gay, 1998). However, this microscale damage accumulates during an intermediate portion of the loading up phase. The eventual global compressive failure occurs by the propagation of this localized compression damage. For instance, Goldstein and Osipenko, FR85 studied the lateral propagation of a compression-induced band of damage 'much narrower than its length' while Bažant and Xiang (1997) studied the lateral propagation of axial splitting cracks. The propagation of these lateral 'compression cracks' have associated energy release rates; an energy-type quasibrittle size effect should be realized. Macroscopic compression failure mechanisms such as the two just discussed are relevant to a certain scale and above and, in all likelihood, do not occur at lab-scale, and will not occur in *in-situ* field experiments even, if they are of the same order in size as the ice sheet thickness.

6.6. *Scaling the fracture of ice*

The question of scale effects on ice properties has long been of interest (Weeks and Assur, FR72). Concerns regarding scale effects on the fracture toughness and tensile strength of ice were raised much later by Dempsey in (FR91) and Dempsey (1996).

The importance of specimen size and cracking orientation versus the predominant *c*-axis orientation was already evident from earlier results obtained by Parsons and Snellen (FR85) and Parsons et al. (FR86). At lab-scale (less than 0.5 m), issues such as inhomogeneity and polycrystallinity are especially important to the fracture behavior of ice. Because of the large grain sizes that one can encounter in ice sheets, it is essential that the effects of sample size on the fracture behavior be determined. In other words, are small-scale (lab-scale) results applicable at larger scales (at the scale of ice-structure interactions, for instance)? In an attempt to answer some of these questions, full thickness lake and sea ice edge-notched plates (Dempsey et al., 1999) were tested over the size range of ($0.34 < L < 28.64$ m) and ($0.5 < L < 80$ m), respectively. The results were rather interesting.

The lab- to structural-scale ($0.34 < L < 28.64$ m) fracture tests conducted on S1 freshwater lake ice at Spray Lakes, Alberta, used the base-edge-notched reverse-tapered plate geometry and covering a size range of 1:81. A Bažant-type size effect analysis of the measured fracture strengths (which do reveal a significant dependence on scale) was unexpectedly clouded by the fact that the data collected violates the associated scatter requirements, even though the size range tested is large. Moreover, via Hillerborg's fictitious crack model, large fracture energies were back-calculated (of order 20 N/m), but for miniscule process zone sizes; in addition, not all of the measured deformations for each test could be matched simultaneously. Apparently, these very warm S1 macrocrystalline lake ice experiments were dominated by nonlocal deformation and energy release rate mechanisms, in all likelihood brought about by grain boundary sliding. The reduced effectiveness of both the Bažant-type size effect analysis and Hillerborg's fictitious crack model (both described in Bažant and Planas, FR98) is due mainly to the lack of crack growth stability achieved in the experiments. These unstable fractures truncated the fracture process. Given the irregular and large grain structure, the very warm ice temperatures, and the non-negligible grain boundary surface energy, there was a marked dependence on specimen size and distinctly non-unique pre-failure processes occurred. These observations have spurred an interest in fracture size effects versus polycrystalline inhomogeneities (Abdel-Tawab and Rodin, 1999; Ballarini et al., 1999). If grain boundary sliding can occur with some inelasticity, apparently one encounters a situation in which the test specimen must incorporate not hundreds but thousands of grains.

The lab- to structural-scale ($0.5 < L < 80$ m) *in-situ* full thickness (1.8 m) fracture tests were conducted on first-year sea ice at Resolute, N.W.T. using self-similar (plan view) edge-cracked square plates. With a size range of 1:160, the data has been used, via size effect analyses, to evaluate the influence of scale effects on the fracture behavior of sea ice over the range 0.1 m (laboratory) to 100 m and to predict the scale effect on tensile strength up to 1000 m. The influence of scale on the ice strength and fracture toughness is dramatic. For the thick first-year sea ice tested, the size-independent fracture toughness is of order $250\ \mathrm{kPa}\sqrt{\mathrm{m}}$, not the $115\ \mathrm{kPa}\sqrt{\mathrm{m}}$ that has been commonly used. The number of grains spanned by the associated test piece was 200, much larger than the number 15 typically quoted for regular tension-compression testing. The size-independent fracture energy was 15 N/m, while the requisite LEFM test size for the edge-cracked square plate geometry (for loading durations of less than 600 s and an average grain size of 1.5 cm), was 3 m square. Size effect analyses of sub-ranges of the data show that unless the specimen sizes tested are themselves sufficiently large, the true nature of the scale effect is not revealed. This was a concern raised by Leicester 25 years ago: '*A particularly dangerous aspect noted of the size effect is that it may not occur unless member sizes are sufficiently large and consequently may not appear in scaled-down laboratory testing*' (Leicester, FR73). In other words, the predictive capability of the various size effect laws showed an hitherto unemphasized dependence on the actual subset used. The goodness of fit was highly dependent on the subset of the size range covering the major area of change. All size effect laws are *ad hoc*, and all can in general be made to fit the data — this matter is pursued in greater detail in Dempsey et al. (1999). In the case of the fracture tests reported in the latter paper, based on the lab-scale and field-scale strength data measured between 0.1 and 3 m and using Bažant's size effect law, it is possible to accurately predict the tensile strengths for all of the remaining tests, up to and including 80 m.

By direct measurements, the fracture scaling of first-year sea ice 1.8 m thick at Resolute, N.W.T. has been established from lab-scale to structural-scale (0.1 to 100 m). During later experiments, the microstructure of sea ice has been studied on the scale of the ice sheet thickness, and the dramatic skeleton sub-structure of the brine drainage networks (Cole and Shapiro, 1998) has been made much clearer. At this stage, one is afforded the following length scales in sea ice: the platelet spacing, grain size, brine channel diameters, brine channel spacing, ice sheet thickness, thermal crack spacing, floe size, aggregate scale, shear band widths, and so on. The spacing of the brine drainage channels is possibly a significant factor in the 3 m transitional size found above. The evolution of this structure with ice thickness needs to be understood, as does the influence of thermal cracks at larger scales (between 100 m and several km). Apparently, thermal cracks are spaced on the order of 200 m apart (Evans, FR71; Gold, FR71). Thermal cracking may weaken large floes (larger than 1 km, especially), or perhaps this is caused more by thickness variations in the thick ice sheet combined with the thin ice of refrozen leads. The expressions deduced by Dempsey (1996) for full thickness tensile strengths via several size effect laws coincide with geophysical strengths at the scale of 1 km. As noted therein, the strengths predicted to occur at the scale of 1 km were predicted to lie between 11 and 38 kPa. For 1.8 m thick ice, this is the same as strengths per unit thickness to lie between 20 and 70 kN/m. Note that Coon et al. (1998) set the unconfined tensile limit (based on a great deal of data) for sea ice at 50 kN/m.

6.7. Time dependent fracture mechanics

The tensile fracture of ice is complicated by the presence of creep, even for short times. The fracture of freshwater ice (Dempsey, in FR91; Weber and Nixon, 1997a; Rist et al., 1999) has been modeled recently by Nanthikesan and Sunder (1995) using the constitutive theory developed by Sunder and Wu, FR89b; Sunder and Wu, FR90. The latter theory does not harbor the ability to predict the onset of crack growth, but does accurately model pre-peak deformations. The time dependent fracture mechanics necessary to describe the tensile fracture of sea ice has now been developed (Mulmule and Dempsey, 1998). The fracture of sea ice has been modeled using a viscoelastic fictitious crack (cohesive zone) model. At the outset, both the stress-separation law active in the cohesive zone and the creep compliance function for the bulk material were unknown and had to be back-calculated through load versus crack opening displacements at the crack mouth, an intermediate location, and at the physical crack tip. The scale-invariant fracture energy (at structural scales, at least) has been determined to be 15 N/m. The scale effect on strength can now be predicted as a function of any crack length and for any geometry. The scale dependence on K_{Ic} (which is used ubiquitously in the ice mechanics literature to be the fracture toughness relevant to the particular model under discussion) can now be evaluated.

6.8. *Slow fracture velocities in sea ice*

During the structural-scale fracture tests (3 m square and 30 m square, 1.8 m thick) conducted at Resolute, N.W.T. in 1993 (Dempsey et al., 1999), the crack growth characteristics were measured acoustically (Xie and Farmer, 1993). It was observed that the initial crack growth included a few infrequent meter-scale jumps, followed ultimately by a series of centimeter-scale, almost continuous, stick-slip failures until a visual fracture formed on the ice surface. Although the small-scale (grain size) crack jumps registered speeds of order 400 m/s, the overall averaged crack velocity was of order 10 m/s. Similar slow crack velocities were measured by Parsons et al. (FR87). In their studies of the fatigue behavior of Antarctic sea ice, Langhorne and Haskell (1998) discovered that the through-the-thickness cracking occurred in discrete steps and was slow. With similar observations being made both in the Arctic and at lab-scale by Petrenko and Gluschenkov (1996), these slow crack velocities are now accepted and await analytical corroboration. The slow velocities observed may well be because of energy being dissipated in the vicinity of the crack tip due to movement of the brine through the extended brine drainage networks. This idea is supported by the fact that faster crack velocities are observed at colder temperatures. An interesting area of research is to now explain these slow crack velocities in sea ice through an appropriate fracture mechanics treatment (a suitable viscoelastic/viscoplastic constitutive sea ice model will be required as well). These slower than expected crack velocities may also be connected with the transitions observed with indentation velocity (Sodhi et al., 1998).

6.9. *Breakthrough ice loads*

In cold regions, floating ice sheets are often used as roads, airfields, parking lots (especially by icefishing enthusiasts), and construction platforms (Gold, FR71; Kerr and Palmer, FR72; Kerr, FR76; Ashton, FR86; Masterson and Smith, in FR91).The topic of moving loads on an ice sheet has been the focus of a recent book by Squire et al. (FR96). The breakthrough load of a floating ice sheet has been an area of recent intense activity; see the recent papers by Bažant and Kim (1998) and Sodhi (1998a) and the references therein. On the basis of plastic limit analysis, as well as small-scale and full-scale experiments, Sodhi (1998b) deduced that the breakthrough load P is proportional to h^2 where h is the ice sheet thickness. Bažant and Kim (1998) approached this problem using fracture mechanics, treating a radial array of part-through cracks. The latter authors assert that there is a thickness (size) effect. Slepyan (1990) deduced that P varies as $h^{13/8}$ via analyses that did not include the effects of crack closure (the wedging or dome effect, as it is also called). Dempsey et al. (1995) showed that crack closure effects cause P to vary as $h^{3/2}$. However, the ultimate load to cause complete penetration of floating ice sheets is higher than that to nucleate the radial crack system. In reality, the radial crack formation is followed by the sequential formation of a number of circumferential cracks, the loads are applied for a considerable duration, and creep deformations may be significant (Fransson, FR85), especially in the case of very warm ice temperatures. In addition to the finding the peak load, there is interest in being able to both model the time dependent deflections and predict the breakthrough event. Particularly from a safety viewpoint, the bearing capacity of time dependent loads on a ice sheet is an important topic.

6.10. *Thermal fracturing*

The modeling and measurement effort related to thermally induced fracturing has obvious relevance at a scale lying between 100 m and above (Evans, FR71; Evans and Untersteiner, FR71). Thermal fracturing is the most pervasive of Arctic ambient noise mechanisms (at frequencies between 500 and 1500 Hz). Generally, it is understood that extensive thermal stressing occurs from early fall through spring in both first year and multiyear ice. However, thermally induced fracturing is apparently more extensive in multiyear ice. Episodes of high thermal fracturing activity occur generally on successive cold nights and following frontal passages (every 3 to 7 days). Thermal fracturing impacts the entire ice pack on almost a daily basis. It has been speculated that thermal forcing extends existing cracks laterally and vertically with the ability to weaken and, eventually, split a floe. As cracks are extended by thermally induced fracturing, a floe's capability to withstand additional tensile stressing is further weakened. Tensile thermal forcing will tend to eventually split a floe along existing cracks and form leads. Moreover, thermal cracks may well have an orientation as a result of the ubiquitous *c*-axis alignment. Thermal fracturing occurs in the ice pack at very low stress levels, on the order of 50 kPa in tension (Richter-Menge and Elder, 1998). A basic difference exists between MY sea ice and FY sea ice (Lewis, 1998). Large-scale tensile strengths appear to be small because of pre-existing cracks on smaller scales. Thermal cracking and isostatic imbalances apparently cause the latter cracks. There is a suspicion that these cracks do not heal completely, if at all, because of high local salinity. Significantly, research that has recorded the size, shape and spacing of these so-called 'pre-existing' cracks has apparently not been done and is needed in order to quantitatively link the 1–100 m fracture/scaling data to the 1 km scale and above. The large scale fracture energy and tensile strength may be used to define a length scale at large scales. Perhaps this length scale ties in with the distance between leads. The next step is to learn if the fracture energy links into a material property that controls the spacing between leads when these regular systems of leads appear. Eventually one would like to link behavior across the scales of 1–10 km, from a single lead to the lead systems observed in satellite imagery.

Bažant (1992) hypothesized that large-scale fractures of sea ice plates in the Arctic could be caused by the release of thermally induced bending moments due to major temperature changes. The critical temperature difference required was found to decrease as (ice sheet thickness)$^{-3/8}$. The previous studies by Evans (FR71), and Evans and Untersteiner (FR71) had estimated that a typical crack spacing of 200 m would be found for the spacing of thermal cracks, which are typically regarded as being like part-through surface cracks, up to tens of meters in length. The degree of consolidation of the Arctic ice floes varies with time, as will the scale on which nature will attempt to release energy via thermal cracking. For this reason, the formation of a few major through-the-thickness large-scale thermal fractures may occasionally occur, although less often (one would suspect) than the numerous part-through thermal cracks.

6.11. *River ice fracture and breakup*

The transverse cracking of river ice covers is a precursor to breakup of the ice cover, which itself may lead to ice jams and floods. There is considerable interest in being able to forecast whether, or when, breakup might occur as a result of an anticipated runoff event. Until recently, estimates of the river ice strength were deemed too low, being of the order of 10–100 kPa. Following up on the scale effects in tensile strength reported by Dempsey (1996), Beltaos (1998) has now provided an explanation for these weak river-ice breakup strengths.

6.12. Wave-ice interaction and sea ice breakup

The wave-induced breakup of intact ice sheets, ice floes, tabular icebergs, and the calving of ice tongues, is an important phenomenon, as illustrated by the premature November 1997 postponement of the Cape Roberts Antarctic Ross Sea Floor Drilling Project. There are several topics at the core of geophysical effects of wave/ice interaction about which very little is known. These include the effect that inhomogeneities (cracks, ridges, changes in thickness, rubble) have on wave propagation; the scattering and damping of waves by pack ice; and the lateral/vertical response of floes in a wave field causing jostling that affects freezing together of floes, damping of the waves, and the herding of floes. Wave propagation can be used to infer the large-scale propagation characteristics of an inhomogeneous sea ice cover, but that idea has only recently been explored and needs development (in November 1998 in McMurdo Sound, Antarctica, C. Fox and T. Haskell measured the surface strain due to local harmonic forcing). For example, the best types of excitation and sensors are still open questions. Questions relating to flexural response of homogeneous fast ice or floes have largely been answered (Wadhams, FR86; Fox and Squire, 1994; Squire et al., FR95; Meylan et al., 1997; Langhorne et al., 1998). Of related interest is work on the fatigue fracture of ice (Weber and Nixon, 1997b; Langhorne and Haskell, 1998).

6.13. Formation and orientation of leads

There is considerable interest not only in the formation of leads but also in their orientations (Marko and Thomson, FR75 Marko and Thomson, FR77; Pritchard, FR88; Coon et al., 1992; Walter and Overland, 1993; Stern et al., 1995; Dempsey, 1996; Hibler and Schulson, 1997; Coon et al., 1998). What scale controls the formation of leads? What is a lead (Lindsay and Rothrock, 1995)? Whatever model one puts forward, actual *in-situ* full thickness fracture experiments (Dempsey et al., FR95) reveal that one has to strongly bias the experiment before a crack will run in any direction other than orthogonal to the preferred *c*-axis direction. While atmospheric forcing may cause lead formation, it is probable that local near-tip tensile fracture processes always control the initiation and propagation of a crack in ice. The prediction of leads and their orientations poses very severe demands, because of sub-scales in both space and time. It seems very logical in the future to make direct use of remote sensing data that will be available day-to-day in a ‘predictor-corrector’ mode to help keep the ice dynamics models tuned to the actual conditions.

6.14. Model basin testing and centrifuge modeling

Model basin testing is based on two approaches: Froude scaling, in which gravitational forces are either dominant or important in comparison to forces induced by any ice failure process, (bending failure of ice on sloping structures, or buckling failure against vertical structures), and non-Froude scaling in which gravitation forces are not significant in comparison to forces induced by ice failure process (crushing and splitting failure of ice against vertical structures). Modeling efforts to predict ice forces on sloping structures and ships have produced good results, because flexure is the primary mode of failure. Besides changing the size, the major requirements of this modeling effort is to reduce the modulus of elasticity and flexural strength of the model ice to bring the ice failure loads in proportion to the gravitational loads. While there is a lot of full-scale data to verify the overall resistance during ship-ice interaction, the same is not true for model tests with structures. Because the interaction is much too complicated to handle theoretically at present, there is a need to do model tests. The state-of-the-art as regards model basin testing has advanced significantly. New model ice has been invented (layered ice with granular and congelation ice) with bubbles interspersed to induce brittle behavior and to have the correct density of ice. The IAHR Working Group on Ice Modeling Materials (Timco, 1992) has reviewed the different types of model ice and their limitations. In general, most mechanical and physical properties can be well scaled, which is often accomplished using special chemical doping and growth techniques. Usually the scale factor for the model test is based on the most important mechanical property of the ice (flexural for ship trials and compressive strength for vertical structures) since not all properties can be scaled simultaneously. The model tests provide good information on overall loads on ships and structures, but they have limitations in understanding of physical processes.

Ship model testing in ice basins is now in much use (Tatinclaux, FR88). The scaling of the resistance to icebreaking, and the division of the resistance into components (such as that due to breaking ice, passage through broken ice, submerging and rotating broken ice, passage through open water) has been attempted by many. A universally accepted method is not yet available (Colbourne and Lever, in FR91, Colbourne and Lever, 1992). No one dimensionless number has gained widespread popularity, and there is no reason to expect one scaling law to apply across the whole range of parameter values. Goldstein and Osipenko, (FR85 in FR91, 1993) have taken a distinctly fracture mechanics approach to the modeling of icebreaking. There are some interesting ideas in these papers, as there was in the earlier paper by Atkins and Caddell (FR74), which introduced fracture toughness and crack velocity into the same dimensionless number. Centrifuge modeling has been used to simulate ice/structure interactions (Barrette et al., 1998). The latter paper reviews applications over the last 15 years, and briefly discusses the unresolved issues of whether centrifuge modeling can be used because of difficulties in controlling the brine content, and scaling incompatibilities if fracture is involved (Palmer, 1991). The mechanics of ridging and rafting has long been of interest (Parmerter and Coon, FR72; Parmerter, FR75). Because it is difficult to observe the formation of ridges in the field, the ice deformation and failure mechanisms active during ridging events are now being both simulated and studied in an ice basin (Tuhkuri and Lensu, 1998; Hopkins et al., 1999).

6.15. *Ice gouging*

Most offshore oil and gas is transported to shore by underwater pipelines. Active ice gouging in the seabed is a prevailing threat in many parts of the world, including the Beaufort Sea, Davis Strait, the Barents and Kara Seas, and the sea offshore Sakhalin. A gouging ice mass may be viewed as an extremely blunt cutting implement, one that pushes the seabed soil downwards and sideways. Until 1990, the accepted consensus was that a pipeline would be safe if it was lowered far enough into the seabed for the deepest expected gouge to pass above the buried pipeline without contact. However, it is now suspected that the deformations beneath a gouge are so severe that a pipeline might be bent to an intolerable extent even though the ice passed above with no actual contact. Additional factors include the pre-existing roughness of the seabed as well as the geotechnical profile underlying the sea bed (Woodworth-Lynas et al., 1996; Palmer, 1997).

6.16. *Ice dynamics models*

Until recently, ice dynamics models have described ice as an isotropic material; grid sizes have been assumed to be sufficiently large to include a representative number of randomly oriented leads and ridges. However, it has become apparent that the formation of leads and ridges is not random: there can appear strongly oriented features and concentrated (linear) shear zones (Pritchard, FR88; Coon et al., 1992; Overland et al., 1995; Stern et al., 1995). The most important research topic in sea ice dynamics at present is the formulation and implementation of physically based models (Coon et al., 1992; Coon et al., 1998; Hibler and Schulson, 1997; Tremblay and Mysak, 1997; Overland et al., 1998; Pritchard, 1998a) that account for the formation and evolution of individual lead systems directly. The objective is to incorporate the behavior of individual leading, rafting and ridging events into the large-scale constitutive behavior of pack ice.

6.17. *Marginal ice zone*

The marginal ice zone (MIZ) is the bordering portion of the pack ice within which the deformations are influenced significantly by ocean waves. The floe sizes are typically smaller, ranging from 10 m to several kilometers. Two constitutive approaches are being researched. The first is a collisional rheology, in which the MIZ is modeled as a large number of discrete solitary floes (Shen et al., FR87; Meylan et al., 1997). The second is a continuum plastic-type rheology (Gutfraind and Savage, 1997; Huang and Savage, 1998). Studying the transition between these two types of rheology has recently begun (Shen and Aidanpää, 1998).

6.18. *River ice dynamics*

The existing constitutive laws used in sea ice dynamics models have limitations in that they can not simulate the complete range of river ice transport and jam processes. Recently, the Hiber-type viscous-plastic rheology was modified to facilitate the description of river ice conditions at very small or zero deformation rates. This new river ice dynamics model has been used to describe lake ice transport processes and river ice jamming (Shen et al., 1997; Liu et al., 1998; Lu and Shen, 1998).

6.19. *Planetary ice mechanics*

On a planetary scale, Venusian tectonics appears to more closely resemble the ridges and leads in a convergent field of Arctic pack ice (Noltimier and Sahagian, 1992) while there is reason to believe that an understanding of sea-ice processes can provide insight into the complex geology of Europa (Greeley et al., 1998).

7. Conclusions

Ice mechanics is an important contributor to a diverse range of scientific investigations and engineering applications, including climate modeling, climate change, heat budget analyses, forecasting ice conditions icebreaking and ship navigation, the opening of the Northern Sea Route, ice loads on fixed and floating structures, ice gouging of the sea floor above buried well completions and pipelines (important for the offshore oil and gas industry), submarine surfacing through ice, aircraft landings on ice, winter river-ice, lake-ice and sea-ice-surface transportation and pollutant transport, oil spills and other environmental issues.

Ice mechanics is of central importance to the quantitative description of ice deformation processes and the role of ice movement over a vast range of differing scales in the atmosphere-ice-ocean system.

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